Calculations Guidance

What is it?

This calculation guidance helps teachers to identify the preferred methods of mental and written calculations for our school. It aims to build on the methods and procedures taught in the infant school.
This guidance contains the key procedures that will be taught within our school.

Why do we need this*?*

It is necessary to ensure consistency and progression of methods for calculation throughout the school. This guidance provides teachers with a clear path of progression in calculation, allowing them to help children progress and identify stages of progression that children are struggling with.

How to use this guidance

The guidance is in 2 sections; addition and subtraction, multiplication and division. Within each section it is divided into stages. National age expectations are highlighted to show the stages children are expected to reach (National Curriculum 2014). We keep expectations high and are looking at above average progression over KS2. When planning for a class it will be necessary to look at the stage below and /or the stage above.

Aims:

Children should be able to choose an appropriate and efficient method of calculation for a given task. Children working in the school at level 4 and above will have been taught a compact standard method for each operation.

General Progression:

- *Establish mental methods based on a good understanding of place value*
- *Use of informal jottings to aid mental calculations*
- *Develop use of empty number line to help mental imagery and aid recording*
- *Use models and images to promote understanding.*
- *Use partitioning and recombining to aid informal methods*
- *Use inverses to check calculations*
- *Introduce expanded methods*
- *Develop expanded methods into compact standard written form*

Once a child has become secure in a particular form they will move on to learn other methods, so that they build a bank of methods. Children will be encouraged to then select an appropriate method to solve problems.

When are the children ready for formal written calculations?

Before formal written methods can be used it is necessary for the children to have a secure understanding of the following.

Addition and subtraction:

- ☺ *To know place value to 3 digits, HTU*
- ☺ *To be able to partition a 3 digit number into hundreds, tens and units.*

- ☺ *To know securely addition and subtraction facts to 20*
- ☺ *To be able to add 3 single digit numbers mentally*
- ☺ *To be able to add and subtract any pair of 2 digit numbers mentally, using a strategy of their choice*
- ☺ *To be able to explain the strategy orally and record it using informal jottings.*

Multiplication and Division:

- ☺ *To know 2, 3, 4, 5, and 10 times tables*
- ☺ *To know the result of multiplying by 0 or 1*
- ☺ *To understand **0 as a place holder***
- ☺ *To be able to multiply two or three digit numbers by 10 and 100*
- ☺ *To be able to double and halve two digit numbers mentally*
- ☺ *To be able to use multiplication facts to derive mentally other multiplication facts*
- ☺ *To be able to explain their mental strategies orally and record them using informal jottings*

Before carrying out a calculation, children will be encouraged to consider some or all of the following points dependent on their current stage. (for example, calculators will start to be introduced in year 5)

- ☺ *Can I do it in my head?*
- ☺ *What size do I expect the answer to be?*
- ☺ *Could I use jottings to keep track of the calculation?*
- ☺ *Do I need to use an expanded or compact written method?*
- ☺ *Do I need a calculator?*

Vocabulary

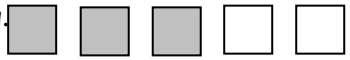
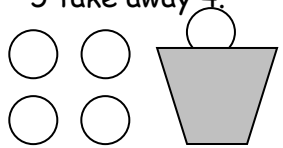
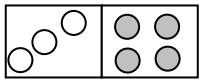
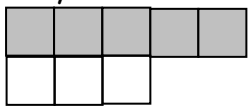
Misconceptions will occur, and need to be rectified quickly. It is necessary to use clear, unambiguous language to minimise misconceptions, and this should be consistent across the whole school.

*When using column methods for subtraction the children should be told to '**exchange**' a ten for units, or a hundred for tens, **not** to 'borrow' a ten or a hundred.*

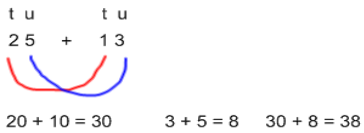
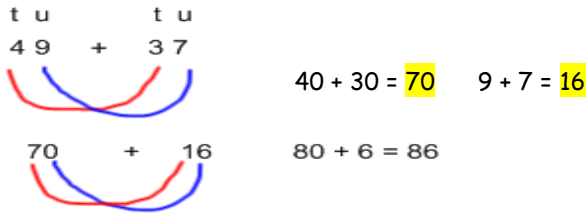
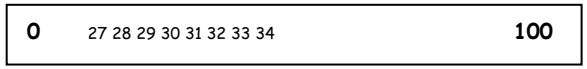
*When teaching multiplication / division by 10, 100 etc., **the figures move, not the decimal point**. The dp always resides between the units and the tenths!*

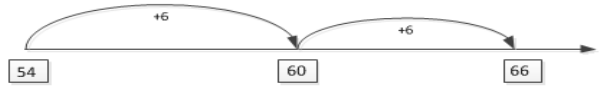
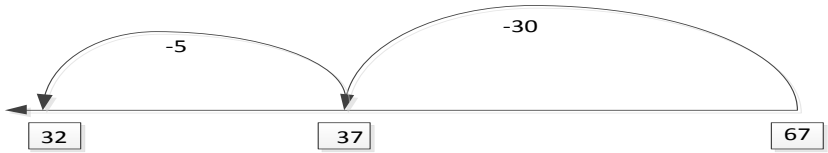
*When multiplying an integer by 10 we **do not ADD a 0.** Adding implies addition and the addition of a number and zero means the original number remains unchanged.*

Contents

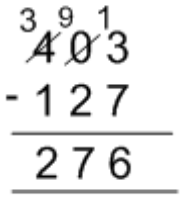
	Addition	Subtraction
	• Use of language: more, less, bigger, smaller, different, same • Begin to relate addition to combining two groups of objects e.g.  Making 5 in different ways • One more; +1	• How many ways can you break up 5? • Begin to understand subtraction as 'taking away' e.g. • 5 take away 4.  How many have I left in the cup? • One less; -1
Stage 1	• Add single digit numbers using fingers • Use materials to add up.  • Use practical resources like dominoes to record number sentences e.g. 3+4=7 or 4+3=7 • Vary position of missing numbers in number sentences. • Develop understanding of addition as counting steps along a number line. • Use number tracks or number lines to record on. (Prepared and children's own.) • Put the largest number first when adding. • Know that addition is commutative. (Not affected by order) e.g. 15 + 4 is the same as 4 + 15.	• Continue to develop vocabulary including 'difference between' and 'how many less is … than …'  • Use pictures and visual aids to record calculations. E.g. how many are left when 3 bricks are taken away? • Use number tracks or number lines and 100 squares. • Vary position of missing numbers in number sentences. • Begin to use and apply the inverse operation e.g. 6 + 5 = 11 11 – 5 = 6 • Find out 'how many more make …?' by counting on (complementary addition) • Count back from any number up to 100 • Subtract one digit numbers from 2 digit numbers

	Addition	Subtraction

Stage 2	• **Count on in 10's from any two digit number**
	• Continue to use practical resources
	• Use 100 square to support counting on.
	  ○ Count on in 1s
	  ○ Count on in 10s (e.g. 60 + 30)
	  ○ Count on in 10s and 1s (e.g. 41 + 24)
	• Begin to **partition** and recombine (seeing 12+15 as 10+10 and 2+5, then 20+7=27).
	• Use washing lines
	 
	
	• And double washing lines
	 
	
	• Addition of three 2 digit numbers adding tens first then units.
	• Know number bonds to 20
	• Use number tracks and empty number lines to support :
	 
	
		• Continue to develop vocabulary of difference, less than, fewer than.
		• Number tracks and empty number lines to count back and begin to count on to find the difference.
		• There are 34 children in the class, 27 go to the hall. How many are left?
		•
		 
		
		• Partition the 2nd number and subtract.
		• E.g. 37 – 12 ⇨ 37 – 10 = 27
		  • 27 – 2 = **25**
		• (NB 'washing lines' don't work for subtraction)
		• Partition numbers in different ways e.g. 27 can be partitioned into 20+7 or 10+17 to support calculations using a 100 square.
		• Use number lines counting back in 10's then units.

	Addition	Subtraction
Stage 3	• *Mental methods include partitioning and compensating.* *E.g. 31 + 17 is 30 + 10 + 7 + 1* *31 + 17 is 30 + 18* • Continue using informal written methods. • Use number lines, particularly empty number lines to model and support addition. • Begin to use expanded written method e.g. 45+17= 40 5 + 10 7 50 + 12 → 62	• *Mental methods include partitioning and compensating.* *E.g. 31 – 17 is 31 – 10 -7* *31 – 17 is 30 – 16* • Use number lines counting in multiples of 10 then units. • Informal counting on e.g.66 – 54 = …  6 + 6 = 12 • Leading onto expanded written methods in a vertical layout: 60 6 -50 4 10 + 2 → 12 • Counting back on a number line e.g. 67 – 35 = 32 

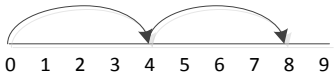
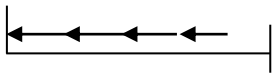
	Addition	Subtraction
Stage 4 *This stage should be reached by the end of year 3 (National Curriculum 2014)*	*Mental methods –regular practise of mental addition including lists of multiple numbers. Explore how children visualise numbers in their heads and encourage them to use a variety of methods for manipulating the numbers.* Expanded written method, vertical layout, adding the **least significant** number first. 2 6 4 + 1 4 8 1 2 1 0 0 3 0 0 4 1 2 Children will progress to using the compact written method, involving carrying, with least significant number first. 2 6 4 + 1 4 8 4 1 2 1 1	*Mental methods –regular practise of mental subtractions. Explore how children visualise numbers in their heads and encourage them to use a variety of methods for manipulating the numbers.* Continue to use empty number lines. Expanded written methods using vertical layout: 81 – 57 = 80 1 → 70 11 50 7 50 7 20 4 = 24 This is a teaching stage Introduce compact written method: e.g. 63 - 27 5 1 6 3 - 2 7 3 6

	Addition	Subtraction
Stage 5	• Continue using mental and informal strategies (number lines work well with time, money, weight etc.) • Extend written methods to column addition of two numbers less than 10,000. By end of year 4 (NC 2014) • Use with decimals e.g. money, length, weight. By end of year 5 (NC 2014) • Extend written methods to column addition of two or more numbers, or decimals in different contexts. By end of year 5 (NC 2014) • Be able to select a suitable strategy (mental, informal, compact, calculator) when solving problems. By end of year 5 (NC 2014)	• Teach calculations with 0 in the tens column e.g. 403 – 127  • Extend to larger numbers and decimal money. By end of year 5 (NC 2014) • Extend written methods for subtraction with decimal numbers with 2 decimal places in different contexts. By end of year 5 (NC 2014) • Children will be able to choose the most efficient and appropriate method for each calculation. By end of year 5 (NC 2014)

	Multiplication	Division
	Vocabulary: Jumps, hops, steps. Use of practical equipment to count in repeated groups of the same size e.g. Make a bead necklace 2red, 2blue, 2 red, 2 blue etc. A pair of socks, gloves, etc.	Share objects into equal groups e.g. share the fruit for a snack; give out one cup to each person.

Stage 1	• Draw pictures to show equal sets: e.g. how many wheels on 3 bikes?
	• Count on and back in twos to 20, fives to 50 and tens to 100
	• Say the tens number that goes before/after a given one.
	• Know doubles of numbers to 10
	• Sort objects into groups to count.
	• Solve practical problems that combine groups of twos, fives and tens.
	  ○ There are 5 sweets in one bag, how many sweets are there in 3 bags?
		• Draw pictures to show sharing and grouping: I have 8 wheels, how many bikes can I make?
		• Solve practical problems sharing groups into twos, fives and tens.
		• 8 shared between 2 people
		  ** **
		  ** **
		  = 4 each
		• Know halves of numbers to 20
		• Recognise odd and even numbers

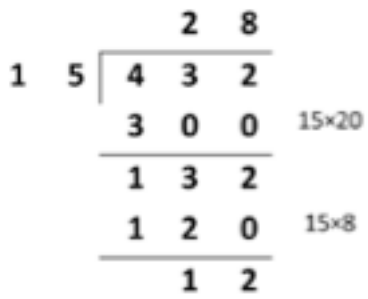
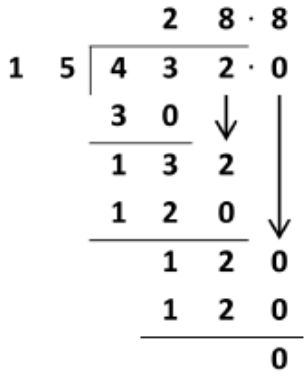
Stage 2	• Count confidently in steps of two to 100, five to 100 and ten from any nº. to 100, and begin to count in steps of 3 and 4.
	• There are 4 apples in one box. How many apples in 6 boxes?
	• Create groups of objects and record as repeated addition and a number sentence
	  Eg 5 x 3 (State as 5 'lots of 3')
	  = 3 + 3 + 3 + 3 + 3
	  = 5 x 3 = 15
	• Use of visual support such as Number tracks, empty number lines, 100 square etc.
		Develop vocabulary involved in division, divided by / between, repeated subtraction, how many groups of … in …
		• Count confidently in steps of two, five and ten.
		• Using practical equipment to share into equal groups
		  e.g. 15 ÷ 3 → * * * * *
		  * * * * *
		  * * * * * = 5

	Multiplication	Division
	4 x 2  2 x 4  • Understand multiplication as repeated addition and making arrays using practical equipment e.g. 3 x 3 = XXX XXX XXX • Double by partitioning 15 x 2 = 30 10 + 5 20 + 10 = 30 • Relate multiplication and division • 2 x 4 = 8 so 8 ÷ 2 = 4	• Relate grouping to arrays and use a number line to illustrate grouping and repeated subtraction e.g. 8÷2= 2 2 2 2  8 - 2 - 2 - 2 - 2 = 4 groups of 2 • Know that dividing by 2 is the same as half • Know that dividing by 4 is the same as quarter • Begin to understand the concept of a remainder
Stage 3	• Know by heart multiplication facts for x2, x3, x4, x5, x8, x10 and recognise multiples of 2, 5 and 10 up to 1000. By end of year 3 (NC 2014) • Multiply a single digit number by x0, x1, x10, x100. • Double any multiple of 5 up to 50. • Derive related facts e.g. 7 x 5, 5 x 7, 35 ÷ 5, 35 ÷ 7. • Use practical apparatus and empty number lines to multiply two digit numbers by a single digit.	Use an empty number line for division e.g.  Divide any 3 digit multiple of 100 by 10 or 100 e.g. 800 ÷ 100 = 8 300 ÷ 10 = 30 Halve any multiple of 10 up to 100 e.g. 50 ÷ 2 = 25.

	Multiplication	Division
Stage 4	• Multiply a 2 digit number by a single digit number, multiplying the tens first and using informal jottings to show the stages of calculation e.g. 4x25 = (4x20) + (4x5) = 80 + 20 = 100 • Develop the extended written method of the grid method to multiply a two digit number by a single digit number e.g. e.g. 37 x 4 **X** | **30** | **7** **4** | 120 | 28 120 + 28 = 148 By end of year 3 (NC 2014) **Note:** Grid method is a **transition** stage towards formal column method.	• Developing and refining written methods for division by dividing a two digit number by a single digit e.g. 84 ÷ 7= 70 + 14 (÷ 7) 10 + 2 = 12 • Use the repeated subtraction or 'chunking' method: By end of year 4 N.B. Chunking not included as statutory requirement in 2014 NC Repeated subtraction – how many sixes in 27? e.g. 27 ÷ 6 = 4 r.3 27 - 6 21 - 6 15 - 6 9 - 6 3 Leading to subtracting larger chunks such as multiples of 10 e.g. 148 ÷4= 148 - 40 (**10**x4) 108 - 40 (**10**x4) 68 - 40 (**10**x4) 28 - 28 (**7**x4) 0 148 ÷ 4 = 10+10+10+7= 37

	Multiplication	Division
Stage 5	Extending written methods, encouraging estimation first. Grid method (HTU x U) e.g. 247 x 7 **1400+280+49=1729** Grid method (TU x TU) e.g. 62 x 36 **2160+72 = 2232** This then leads to a compact written method for multiplication e.g. 2 4 6 x 7 4 2 (6 X 7) 2 8 0 (40 X 7) 1 4 0 0 (200 X 7) 1 7 2 2 *The annotations would be modelled for the children, not expected of them.* 246 x 7 1722 3 4 By end of year 4 (NC 2014)	Extending written methods, encouraging estimation first. Children continue to use the 'chunking' method before progressing to the short division method with repeated subtraction e.g. 196 ÷ 6. 32 r. 4 6) 196 - 60 (**10**x6) 136 - 60 (**10**x6) 76 - 60 (**10**x6) 16 - 12 (**2**x6) 4 *The annotations would be modelled for the children, not expected of them.* This then contracts to the more compact short division method e.g. 32 r. 4 6) 196 By end of year 5 (NC 2014)

	Multiplication	Division
Stage 6	Long multiplication to be applied to problem solving including measures, distance, weight, capacity and money and fractions. To be practiced throughout year 5 and 6 e.g. Farmer Jones wants to fence in a square field. Each side is 118.35 m. How much fence does he need? 118.35 x 4 473.40 m 3 1 2 e.g. A group of 17 people eat in a restaurant. The menu lists the meal price at £23.58 a head. How much is the total cost of the meal?	Children will progress to using the long division method. They will solve problems to find a remainder, show the remainder as a fraction, or find the answer as a decimal. By end of year 6 (NC 2014) **Long division** 432 ÷ 15 becomes 2 8 r 12 1 5 | 4 3 2 3 0 0 1 3 2 1 2 0 1 2 Answer: 28 remainder 12

		432 ÷ 15 becomes  $\frac{12}{15} = \frac{4}{5}$ Answer: $28\frac{4}{5}$ 432 ÷ 15 becomes  Answer: 28·8